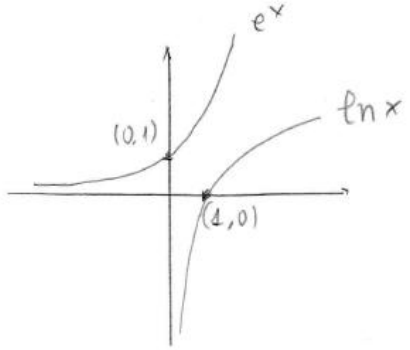
Θέμα Α

A1) απόδειξη σχολικό σελ. 175

A2) $f(x) = e^x$ | $g(x) = \ln x$

$A_f = \mathbb{R}$ | $A_g = (0, +\infty)$

$\Sigma.T. = (0, +\infty)$ | $\Sigma.T. = \mathbb{R}$



$e^x > \ln x$

$e^{2021} > \ln 2021$ άρα $f(2021) > g(2021)$

A3) α) Λ β) Λ γ) Σ δ) Λ ε) Λ

Θέμα Β

$f(x) = 2\sigma\upsilon\nu 2x$

α) $max = 2$

$min = -2$

$T = \frac{2\pi}{2} = \pi$

β) $f(x) = 1$ στο $[0, 2\pi]$

$2\sigma\upsilon\nu 2x = 1$

$\sigma\upsilon\nu 2x = \frac{1}{2}$

$\sigma\upsilon\nu 2x = \sigma\upsilon\nu\frac{\pi}{3}$

$2x = 2k\pi \pm \frac{\pi}{3}$

$x = k\pi \pm \frac{\pi}{6}$, $k \in \mathbb{Z}$

$0 \le x \le 2\pi \iff 0 \le k\pi + \frac{\pi}{6} \le 2\pi \iff -\frac{\pi}{6} \le k\pi \le 2\pi - \frac{\pi}{6}$

$$\left.\begin{matrix} -\frac{\pi}{6} \le k\pi \le \frac{11\pi}{6} \iff -\frac{1}{6} \le k \le \frac{11}{6} \\ k \in \mathbb{Z} \end{matrix}\right\} \begin{matrix} k=0 \text{ άρα } x = \frac{\pi}{6} \\ k=1 \text{ άρα } x = \pi + \frac{\pi}{6} \\ x = \frac{7\pi}{6} \end{matrix}$$

Για $x = k\pi - \frac{\pi}{6}$, ομοίως.

γ) $f\left(\frac{\pi}{12}\right) = 2συν\, 2\frac{\pi}{12} = 2συν\frac{\pi}{6} = 2\frac{\sqrt{3}}{2} = \sqrt{3}$

$f\left(\frac{5\pi}{12}\right) = 2συν 2\cdot\frac{5\pi}{12} = 2συν\frac{5\pi}{6} = 2συν\left(\pi - \frac{\pi}{6}\right) = 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$

$f\left(\frac{\pi}{6}\right) = 2συν 2\frac{\pi}{6} = 2συν\frac{\pi}{3} = 2\cdot\frac{1}{2} = 1$

$f\left(\frac{\pi}{4}\right) = 2συν 2\frac{\pi}{4} = 2συν\frac{\pi}{2} = 0$

Άρα $K = \frac{\sqrt{3}(-\sqrt{3}) + 1}{1-0} = -3+1 = -2$

B2) $2συν^3x + 7ημ^2x + 2συνx - 4 = 0$

$2συν^3x + 7(1 - συν^2x) + 2συνx - 4 = 0$

$2συν^3x - 7συν^2x + 2συνx + 3 = 0$ θέτω $\boxed{w = συνx}$

$2w^3 - 7w^2 + 2w + 3 = 0$ $\quad \pm 1, \pm 3$

<u>Horner:</u>

2	-7	2	3	$ρ = 1$
↓	2	-5	-3	
2	-5	-3	0	

$(w-1)(2w^2 - 5w - 3) = 0 \iff w = 1$ ή $2w^2 - 5w - 3 = 0$

$\Delta = 25 + 24 = 49$

$w_{1,2} = \frac{5 \pm 7}{4} \begin{cases} 3 & \underline{απορ} \\ -\frac{1}{2} & \end{cases}$

Άρα $συνx = 1$ ή $συνx = -\frac{1}{2}$

$συνx = συν0$ $\quad$ $συνx = -συν\frac{\pi}{3}$

$x = 2k\pi$ $\quad$ $συνx = συν\left(\pi - \frac{\pi}{3}\right)$

$k \in \mathbb{Z}$ $\quad$ $συνx = συν\frac{2\pi}{3}$

$x = 2k\pi \pm \frac{2\pi}{3}$, $k \in \mathbb{Z}$

Θέμα Γ

Γ1) $f(x) = x^3 + \alpha x^2 + \beta x + \gamma$

$f(-2) = 24 \iff -8 + 4\alpha - 2\beta + \gamma = 24 \overset{①}{\iff} 4\alpha - 2\beta = 24 \iff 2\alpha - \beta = 12$ ②

$f(0) = 8 \iff \gamma = 8$ ①

$f(1) = 0 \iff 1 + \alpha + \beta + 8 = 0 \iff \alpha + \beta = -9$ ③

Από (2) και (3): $\left.\begin{array}{l} 2\alpha - \beta = 12 \\ \alpha + \beta = -9 \end{array}\right\}$ $\begin{array}{l} \alpha = 1 \\ \beta = -10 \end{array}$

Άρα $f(x) = x^3 + x^2 - 10x + 8$

Γ2) α) $f(x) = 0 \iff x^3 + x^2 - 10x + 8 = 0$

Horner:

$$\begin{array}{cccc|l} 1 & 1 & -10 & 8 & \rho = 1 \\ \downarrow & 1 & 2 & -8 & \\ 1 & 2 & -8 & 0 & \end{array}$$

$(x-1)(x^2 + 2x - 8) = 0 \iff x = 1$ ή $x^2 + 2x - 8 = 0$

$\Delta = 4 + 32 = 36$

$x_{1,2} = \frac{-2 \pm 6}{2} \left\langle \begin{array}{l} -4 \\ 2 \end{array} \right.$

β) $f(x) < 0 \iff (x-1)(x^2 + 2x - 8) < 0$

x	$-\infty$ -4 1 2 $+\infty$
$x-1$	$-$ $-$ 0 $+$ $+$
x^2+2x-8	$+$ 0 $-$ $-$ 0 $+$
$f(x)$	$-$ 0 $+$ 0 $-$ 0 $+$

$x \in (-\infty, -4) \cup (1, 2)$

$f(-x) = -x^3 + x^2 + 10x + 8$

Γ3) $\frac{x+4}{f(x)} \le \frac{2}{f(x) + f(-x) - 18} \iff \frac{x+4}{(x-1)(x+4)(x-2)} \le \frac{2}{2x^2 + 16 - 18}$

$\frac{1}{(x-1)(x-2)} \le \frac{2}{2(x^2-1)} \iff \frac{1}{(x-1)(x-2)} - \frac{1}{(x-1)(x+1)} \le 0$

$(x \neq -1, 1, 2, -4)$ $\iff \frac{x+1-(x-2)}{(x-1)(x+1)(x-2)} \le 0 \iff \frac{3}{(x-1)(x+1)(x-2)} \le 0$

x	$-\infty$		-4		-1		1		2		$+\infty$
x^2-1		$+$		$+$	0	$-$	0	$+$		$+$	
$x-2$		$-$		$-$		$-$		$-$	0	$+$	
$P(x)$		$-$	$\cancel{0}$	$-$	$\cancel{0}$	$+$	$\cancel{0}$	$-$	$\cancel{0}$	$+$	

$x\in(-\infty,-4)\cup(-4,-1)\cup(1,2)$

Θέμα Δ

$f(x)=\ln(e^{2x}-2e^x+3)\quad,\quad g(x)=\ln3+\ln(e^x-1)$

Δ1) $e^{2x}-2e^x+3>0 \overset{w=e^x}{\iff}$ $w^2-2w+3>0$

$\Delta=4-12<0$

$A_f=\mathbb{R}$

$e^x-1>0\iff e^x>e^0 \overset{\nearrow}{\iff} x>0$

$A_g=(0,+\infty)$

Δ2) $f(x)=g(x)\iff \ln(e^{2x}-2e^x+3)=\ln3(e^x-1)\overset{1-1}{\iff}$ πρέπει $x>0$

$e^{2x}-2e^x+3=3e^x-3\iff e^{2x}-5e^x+6=0$

θέτω $w=e^x$, $w^2-5w+6=0$, $\Delta=1$. $w_{1,2}=\dfrac{5\pm1}{2}$

άρα $e^x=3$ ή $e^x=2\overset{1-1}{\iff}$

$\ln e^x=\ln3$ $\quad$ $\ln e^x=\ln2$

$x=\ln3$ $\quad$ $x=\ln2$ Δεκτή

Δ3) $f(x)>2g(x)\iff \ln(e^{2x}-2e^x+3)>2\ln3(e^x-1)\iff$

$\ln(e^{2x}-2e^x+3)>\ln9(e^x-1)^2 \overset{\nearrow}{\iff} e^{2x}-2e^x+3>9(e^{2x}-2e^x+1)\iff$

$8e^{2x}-16e^x+6<0\iff 4e^{2x}-8e^x+3<0 \overset{w=e^x}{\iff} 4w^2-8w+3<0$

$\Delta=64-48=16$, $w_{1,2}=\dfrac{8\pm4}{8}$ ⟨ $12/8=3/2$, $\dfrac{1}{2}$

Άρα $e^x=\dfrac{1}{2}$ ή $e^x=3/2$

$x=\ln 1/2<0$ $\quad$ $x=\ln3/2$ Δεκτή

απορρ.